CHAPTER 30

Differential Equations

Exercise

1. If $y = 2 + c \cdot e^{-2x^2}$, then

(a) $\frac{dy}{dx} = 8x$ (b) $\frac{dy}{dx} + 4xy = 8x$

(c) $\frac{dy}{dx} + 4xy = 0$ (d) $\frac{dy}{dx} - 4xy = 0$

2. If the differential equation $\frac{dy}{dx} = \frac{3y}{2x}$ represents a family of hyperbolas, then the eccentricity is

(a) $\sqrt{\frac{5}{3}}$ (b) $\sqrt{\frac{3}{5}}$

(c) $\sqrt{\frac{5}{2}}$ (d) $\sqrt{\frac{3}{2}}$

3. Solution of $\frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2y^2}$ is

(a) $\sin^{-1} y = \sin^{-1} x + c$

(b) $\sin^{-1} y = \frac{1}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x + c$

(c) $\sin^{-1} y = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{4}\cos^{-1} x + c$

(d) $\sin^{-1} y = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x + c$

4. The degree of differential equation

$\frac{dy}{dx} - x = \left(y - x\frac{dy}{dx} \right)^{-4}$ is

(a) 1 (b) 2

(c) 3 (d) 5

5. A general solution of the equation $\frac{dy}{dx} = y$ is given by

(a) $y = ae^x$ (b) $y = e^{ax}$

(c) $y = e^x$ (d) $y = e^{-ax}$

6. The differential equation of all non-vertical lines in a plane is

(a) $\frac{d^2y}{dx^2} = 0$ (b) $\frac{d^2x}{dy^2} = 0$

(c) $\frac{dy}{dx} = 0$ (d) $\frac{dx}{dy} = 0$

7. The general solution of the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$ is

(a) $x^3 - y^3 = c$ (b) $x^3 + y^3 = c$

(c) $x^2 + y^2 = c$ (d) $x^2 - y^2 = c$

8. The differential equation of all conics whose centre lie at the origin is of order

(a) 2 (b) 3

(c) 4 (d) None of these

9. The differential equation of the family of curves $y = Ae^{3x} + Be^{5x}$, where A and B are arbitrary constants, is

(a) $\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 15y = 0$ (b) $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$

(c) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0$ (d) None of these

10. The solution of the equation $x\frac{dy}{dx} = y + x\tan\frac{y}{x}$ is

(a) $\sin\frac{x}{y} = cx$ (b) $\sin\frac{y}{x} = cx$

(c) $\sin\frac{x}{y} = cy$ (d) $\sin\frac{y}{x} = cy$

11. The differential equation of all circles which pass through origin and whose centres lies on Y-axis is

(a) $(x^2 - y^2)\frac{dy}{dx} - 2xy = 0$

(b) $(x^2 - y^2)\frac{dy}{dx} + 2xy = 0$

(c) $(x^2 - y^2)\frac{dy}{dx} - xy = 0$

(d) $(x^2 - y^2)\frac{dy}{dx} + xy = 0$

12. If $x\frac{dy}{dx} = y(\log y - \log x + 1)$, then solution of the equation is

(a) $\log\frac{x}{y} = ky$ (b) $\log\frac{y}{x} = ky$

(c) $\log\frac{x}{y} = kx$ (d) $\log\frac{y}{x} = kx$

13. The solution of the differential equation $(x^2 - yx^2)\, dy + (y^2 + x^2 y^2)\, dx = 0$ is

(a) $\frac{1}{x} + y + \log y = c$ (b) $\frac{1}{x} + y + \frac{1}{y^2} = c$

(c) $\frac{1}{x} + \frac{1}{y} + \log y - x = c$ (d) None of these

14. The solution of the equation $\frac{dy}{dx} = \frac{y^2 - y - 2}{x^2 + 2x - 3}$ is

(a) $\frac{1}{3}\log\left|\frac{y-2}{y+1}\right| = \frac{1}{4}\log\left|\frac{x-1}{x+3}\right| + c$

(b) $\frac{1}{3}\log\left|\frac{y+1}{y-2}\right| = \frac{1}{4}\log\left|\frac{x+3}{x-1}\right| + c$

(c) $\frac{1}{4}\log\left|\frac{y+1}{y-2}\right| = \frac{1}{3}\log\left|\frac{x+3}{x-1}\right| + c$

(d) None of the above

15. General solution of $\frac{dy}{dx} = \frac{ax+h}{by+k}$ represents a parabola when

(a) $a = 0, b = 0$ (b) $a = 1, b = 2$

(c) $a = 0, b \neq 0$ (d) $a = 2, b = 1$

16. The degree of the differential equation

$$\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0 \text{ is}$$

(a) 1 (b) 2

(c) 3 (d) None of these

17. The solution of differential equation $\frac{dy}{dx} + \frac{y}{x} = e^x$ is

(a) $xy = e^x(x + 1) + c$ (b) $xy = e^x + c$

(c) $xy = c$ (d) $xy = e^x(x - 1) + c$

18. The general solution of $\frac{dy}{dx} = \sin x - y$ is

(a) $y = ce^{-2x} + \frac{1}{4}\sin x - \frac{1}{2}\cos x$

(b) $y = ce^{-x} + \frac{1}{2}\sin x - \frac{1}{2}\cos x$

(c) $y = ce^{-3x} + \sin x$ (d) $y = ce^{-x}$

19. The solution of the equation $\frac{dy}{dx} + 2y = \sin x$ is

(a) $y = 5(2\sin x - \cos x) + ce^{-2x}$

(b) $y = (2\sin x - \cos x) + ce^{-x}$

(c) $y = (2\cos x - \sin x) + ce^{-2x}$

(d) $y = \frac{1}{5}(2\sin x - \cos x) + ce^{-2x}$

20. The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$ is

(a) $(x - 2) = ke^{\tan^{-1} y}$

(b) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

(c) $xe^{\tan^{-1} y} = \tan^{-1} y + k$

(d) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$

21. Solution of differential equation $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$ is equal to

(a) $\log\left(2 + \sec\frac{x+y}{2}\right) = x + c$

(b) $\log\{1 + \tan(x + y)\} = y + c$

(c) $\log\left\{1 + \tan\frac{x+y}{2}\right\} = y + c$

(d) $\log\left\{1 + \tan\frac{x+y}{2}\right\} = x + c$

22. The solution of the equation $(x^2 - xy)\, dy = (xy + y^2)\, dx$ is

(a) $xy = ce^{-y/x}$ (b) $xy = ce^{-x/y}$

(c) $yx^2 = ce^{1/x}$ (d) None of these

23. Solution of differential equation $y\, dx - x\, dy + y^2 x^2\, dx = 0$ is equal to

(a) $3x + x^3y = ky$ (b) $3y + y^3x = ky$

(c) $3y + y^3x = kx$ (d) None of these

24. Solution of the differential equation $(1 + y^2)\, dx = (\tan^{-1}y - x)\, dy$ is

(a) $xe^{\tan^{-1} y} = (1 - \tan^{-1} y)e^{\tan^{-1} y} + c$

(b) $xe^{\tan^{-1} y} = (\tan^{-1} y + 1)e^{\tan^{-1} y} + c$

(c) $x = \tan^{-1} y - 1 + ce^{-\tan^{-1} y}$

(d) None of the above

25. The solution of the equation $x^2\frac{dy}{dx} = x^2 + xy + y^2$ is

(a) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + c$ (b) $\tan^{-1}\left(\frac{x}{y}\right) = \log x + c$

(c) $\tan^{-1}\left(\frac{x}{y}\right) = \log y + c$ (d) $\tan^{-1}\left(\frac{y}{x}\right) = \log y + c$

26. The solution curve of $\frac{dy}{dx} = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}$, $y(-1) = 1$ is
    (a) straight line (b) circle
    (c) parabola (d) ellipse
27. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitrary constants is of
    (a) 1st order, 2nd degree
    (b) 1st order, 1st degree
    (c) 2nd order and 1st degree
    (d) 2nd order and 2nd degree
28. Solution of the differential equation $x^2\, dy - y\,(x + y)\, dx = 0$ is
    (a) $x = ke^{-x/y}$ (b) $x = ke^{x/y}$
    (c) $y = ke^{x/y}$ (d) $y = ke^{-x/y}$
29. The equation of the curve through the point (1, 0) whose slope is $\frac{y-1}{x^2+x}$ is
    (a) $(y - 1)(x + 1) + 2x = 0$
    (b) $2x\,(y - 1) + x + 1 = 0$
    (c) $x\,(y - 1)(x + 1) + 2 = 0$
    (d) $x\,(y + 1) + y\,(x + 1) = 0$
30. The solution of the differential equation $y\, dx + (x + x^2 y)\, dy = 0$ is
    (a) $-\frac{1}{xy} = c$ (b) $-\frac{1}{xy} + \log y = c$
    (c) $\frac{1}{xy} + \log y = c$ (d) $\log y = cx$

ANSWERS

1.	(d)	**2.**	(a)	**3.**	(d)	**4.**	(d)	**5.**	(a)	**6.**	(a)	**7.**	(a)	**8.**	(b)	**9.**	(b)	**10.**	(b)
11.	(a)	**12.**	(d)	**13.**	(c)	**14.**	(a)	**15.**	(c)	**16.**	(b)	**17.**	(d)	**18.**	(b)	**19.**	(d)	**20.**	(b)
21.	(d)	**22.**	(b)	**23.**	(a)	**24.**	(c)	**25.**	(a)	**26.**	(a)	**27.**	(d)	**28.**	(a)	**29.**	(a)	**30.**	(b)

Explanations

1. (d) $y = 2 + c \cdot e^{-2x^2}$

$$\frac{dy}{dx} = ce^{-2x^2}(-4x) = -4c \cdot 4e^{-2x^2}$$

$$= -4x(y-2) = -4xy + 8x$$

or $\frac{dy}{dx} + 4xy = 8x$

2. (a) $\frac{dx}{dy} = \frac{3y}{2x}$

$\Rightarrow \int 2x\, dx = \int 3y\, dy$

$\Rightarrow x^2 = \frac{3}{2}y^2 + c$

$\Rightarrow 2x^2 - 3y^2 = 2c$

or $\frac{x^2}{3} - \frac{y^2}{2} = \frac{c}{3}$

$\Rightarrow \frac{x^2}{c} - \frac{y^2}{\frac{2c}{3}} = 1$

Here, $a^2 = c$ and $b^2 = \frac{2}{3}c$

So, eccentricity $e = \sqrt{\frac{a^2 + b^2}{a^2}} = \sqrt{\frac{5}{3}}$

3. (d) $\frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2 y^2}$

$\Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = \int \sqrt{1-x^2}\, dx$

$\Rightarrow \sin^{-1} y = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x + c$

4. (d) $\frac{dy}{dx} - x = \left(y - x\frac{dy}{dx}\right)^{-4}$

$\Rightarrow \left(\frac{dy}{dx} - x\right)\left(y - x\frac{dy}{dx}\right)^4 = 1$

Degree = 5

5. (a) $\frac{dy}{dx} = y \Rightarrow \int \frac{dy}{y} = \int dx$

$\log y = x + c$

$y = e^x \cdot e^c$ or $y = ae^x$.

6. (a) The general equation of all non-vertical lines in a plane is $ax + by = 1$, where $b \neq 0$.

Now, $ax + by = 1$

$$\Rightarrow a + b\frac{dy}{dx} = 0 \Rightarrow b\frac{d^2y}{dx^2} = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} = 0$$

7. (a) $\frac{dy}{dx} = \frac{x^2}{y^2}$

$\Rightarrow y^2\,dy = x^2\,dx$

$\Rightarrow y^3 - x^3 = -c$

or $x^3 - y^3 = c$ (on integrating)

8. (b) The general equation of all conics whose centre lies at the origin is

$$ax^2 + 2hxy + by^2 = 1$$

Clearly, it has three arbitrary constant, so the differential equation will be of order 3.

9. (b) $y = Ae^{3x} + Be^{5x}$

$y_1 = 3Ae^{3x} + 5Be^{5x}$

$y_2 = 9Ae^{3x} + 25Be^{5x}$

Eliminating A and B from the above equation we get

$$\begin{vmatrix} e^{3x} & e^{5x} & -y \\ 3e^{3x} & 5e^{5x} & -y_1 \\ 9e^{3x} & 25e^{5x} & -y_2 \end{vmatrix} = 0$$

or $$-e^{3x}\cdot e^{5x}\begin{vmatrix} 1 & 1 & y \\ 3 & 5 & y_1 \\ 9 & 25 & y_2 \end{vmatrix} = 0$$

On solving,

$$30y - 16y_1 + 2y_2 = 0$$

or $$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$$

10. (b) Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow x\left[v + x\frac{dv}{dx}\right] = vx + x\tan\frac{vx}{x}$$

$$\Rightarrow v + x\frac{dv}{dx} = v + \tan v$$

or $\cot v\,dv = \frac{dx}{x}$

On integrating

$$\log \sin v = \log x + \log c$$

or $\sin\left(\frac{y}{x}\right) = cx$

11. (a) If $(0, k)$ be the centre on Y-axis, then its radius will be k as it passes through origin. Hence, its equation is

$$x^2 + (y - k)^2 = k^2$$

or $x^2 + y^2 = 2ky$ ...(i)

$$\therefore\ 2x + 2y\frac{dy}{dx} = 2k\frac{dy}{dx} = \frac{x^2 + y^2}{y}\frac{dy}{dx}\ \{\text{From eq.(i)}\}$$

$$\Rightarrow 2xy = (x^2 + y^2 - 2y^2)\frac{dy}{dx}$$

$$\Rightarrow (x^2 - y^2)\frac{dy}{dx} - 2xy = 0$$

12. (d) $\frac{dy}{dx} = \frac{y}{x}\left[\log\frac{y}{x} + 1\right]$

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = v\log v + v$$

$$\frac{dv}{v\log v} = \frac{dx}{x}$$

$\Rightarrow \log(\log v) = \log x + \log k$

$$\Rightarrow \log\frac{y}{x} = kx$$

13. (c) $x^2(1 - y)\,dy + y^2(1 + x^2)\,dx = 0$

$$\frac{1-y}{y^2}dy + \left(\frac{1+x^2}{x^2}\right)dx = 0$$

$$\left(\frac{1}{y^2} - \frac{1}{y}\right)dy + \left(\frac{1}{x^2} + 1\right)dx = 0$$

On integrating, we get

$$\frac{1}{x} + \frac{1}{y} + \log y - x = c$$

14. (a) $\frac{dy}{y^2 - y - 2} = \frac{dx}{x^2 + 2x - 3}$

On integrating,

$$\int\frac{dy}{(y-2)(y+1)} = \int\frac{dx}{(x+3)(x-1)}$$

$$\int\left[\frac{1}{3(y-2)} - \frac{1}{3(y+1)}\right]dy$$

$$= \int\left[\frac{1}{4(x-1)} - \frac{1}{4(x+3)}\right]dx$$

$$\Rightarrow \frac{1}{3}\log\left|\frac{y-2}{y+1}\right| = \frac{1}{4}\log\left|\frac{x-1}{x+3}\right| + c$$

15. (c) $\frac{dy}{dx} = \frac{ax+h}{by+k}$

$(by + k)\, dy = (ax + h)\, dx$

On integrating,

$\frac{by^2}{2} + ky = \frac{ax^2}{2} + hx$

or $\left(y^2 + \frac{2k}{b}y\right) = \frac{1}{b}(x^2 a + 2hx)$

Clearly, if $a = 0$, $b \neq 0$, the equation represents a parabola.

16. (b) $\left(\frac{d^3y}{dx^3}\right)^{2/3} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$

$\Rightarrow \left(\frac{d^3y}{dx^3}\right)^2 = \left(3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4\right)^3$

So, degree = 2

17. (d) $\frac{dy}{dx} + \frac{y}{x} = e^x$

Here, $P = \frac{1}{x}$ and $Q = e^x$

Now, $\text{IF} = e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

$y\,(\text{IF}) = \int Q\,(\text{IF})\, dx + c$

$yx = \int e^x x\, dx + c$

$\Rightarrow \quad xy = xe^x - e^x + c \Rightarrow xy = e^x(x-1) + c$

18. (b) $\frac{dy}{dx} + y = \sin x$

Then, $P = 1$, $Q = \sin x$

$\text{IF} = e^{\int P\, dx} = e^x$

$\Rightarrow \quad y\,(\text{IF}) = \int Q\,(\text{IF})\, dx + c$

$ye^x = \int \sin x \cdot e^x dx + c$

Let $I = \int \sin x \cdot e^x \; dx$

$I = \sin x \cdot e^x - [e^x \cos x + \int \sin x\, e^x dx]$

$= \sin x \cdot e^x - e^x \cos x + \int e^x \sin x\, dx$

$\Rightarrow \quad 2I = (\sin x - \cos x)e^x$

$I = \frac{e^x}{2}(\sin x - \cos x)$

Thus, the solution is given by

$ye^x = \frac{e^x}{2}(\sin x - \cos x) + c$

$y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + ce^{-x}$

19. (d) $\frac{dy}{dx} + 2y = \sin x$

$P = 2$ and $Q = \sin x$

$\text{IF} = e^{\int P dx} = e^{\int 2dx} = e^{2x}$

Solution is

$y \cdot \text{IF} = \int Q \cdot \text{IF}\; dx + c$

$ye^{2x} = \int e^{2x} \sin x\; dx + c$

$ye^{2x} = \frac{e^{2x}}{5}(2\sin x - \cos x) + c$

or $\quad y = \frac{1}{5}(2\sin x - \cos x) + ce^{-2x}$

20. (b) $(1+y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$

$(1+y^2)\frac{dx}{dy} + x = e^{\tan^{-1} y}$

or $\quad \frac{dx}{dy} + \left(\frac{1}{1+y^2}\right)x = \frac{e^{\tan^{-1} y}}{1+y^2}$

$\text{IF} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$

Solution is $x \cdot e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2} \cdot e^{\tan^{-1} y} dy$

or $\quad x \cdot e^{\tan^{-1} y} = \frac{(e^{\tan^{-1} y})^2}{2} + \frac{k}{2}$

or $\quad 2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$

21. (d) $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$ ...(i)

Let $x + y = v \Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 1$

Put in eq (i),

$\frac{dv}{dx} - 1 = \sin v + \cos v$

or $\quad \frac{dv}{1 + \sin v + \cos v} = dx$

$\Rightarrow \quad \frac{dv}{2\cos^2 \frac{v}{2} + 2\sin\frac{v}{2}\cos\frac{v}{2}} = dx$

$\Rightarrow \quad \frac{\frac{1}{2}\sec^2 \frac{v}{2} dv}{1 + \tan\frac{v}{2}} = dx$

On integrating,

$$\log\left(1+\tan\frac{v}{2}\right)=x+c$$

$$\Rightarrow \log\left\{1+\tan\left(\frac{x+y}{2}\right)\right\}=x+c$$

22. (b) $(x^2-xy)\,dy=(xy+y^2)\,dx$

$$\frac{dy}{dx}=\frac{xy+y^2}{x^2-xy} \quad \text{...(i)}$$

Let $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

Put in eq. (i),

$$v+x\frac{dv}{dx}=\frac{vx^2(1+v)}{x^2(1-v)}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{v(1+v)}{1-v}$$

$$\Rightarrow x\frac{dv}{dx}=\frac{v(1+v)}{1-v}-v=\frac{2v^2}{1-v}$$

$$\Rightarrow \int\left(\frac{1-v}{v^2}\right)dv=\int 2\frac{dx}{x}$$

$$\Rightarrow \int\left(\frac{1}{v^2}-\frac{1}{v}\right)dv=2\int\frac{dx}{x}$$

$$\Rightarrow \frac{-1}{v}-\log v=2\log x-\log c$$

$$\Rightarrow -\frac{x}{y}-\log\frac{y}{x}=2\log x-\log c$$

$$\Rightarrow -\frac{x}{y}=\log x^2\cdot\frac{y}{x}=\log xy-\log c$$

$$\Rightarrow xy=ce^{-x/y}$$

23. (a) $y\,dx-x\,dy+y^2x^2\,dx=0$

$$\Rightarrow \frac{y\,dx-x\,dy}{y^2}=-x^2dx$$

On integrating,

$$\frac{x}{y}=-\frac{x^3}{3}+k$$

or $3x+x^3y=ky$

24. (c) $(1+y^2)\,dx=(\tan^{-1}y-x)\,dy$

$$\Rightarrow \frac{dx}{dy}=\frac{\tan^{-1}y-x}{1+y^2}$$

$$\frac{dx}{dy}+\frac{1}{1+y^2}x=\frac{\tan^{-1}y}{1+y^2}$$

$$\text{IF}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$$

Solution is,

$$x\cdot e^{\tan^{-1}y}=\int e^{\tan^{-1}y}\cdot\frac{\tan^{-1}y}{1+y^2}dy+c$$

$$xe^{\tan^{-1}y}=e^{\tan^{-1}y}(\tan^{-1}y-1)+c$$

or $x=\tan^{-1}y-1+ce^{-\tan^{-1}y}$

25. (a) $\frac{dy}{dx}=\frac{x^2+xy+y^2}{x^2}$ ...(i)

Let $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

Put in eq. (i),

$$v+x\frac{dv}{dx}=1+v+v^2$$

or $\int\frac{dv}{1+v^2}=\int\frac{dx}{x}$

$\tan^{-1}v=\log x+c$

$\tan^{-1}\frac{y}{x}=\log x+c$

26. (a) $\frac{dy}{dx}=\frac{y^2-2xy-x^2}{y^2+2xy-x^2}$ ...(i)

Let $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

Put in eq. (i),

$$\Rightarrow v+x\frac{dv}{dx}=\frac{v^2-2v-1}{v^2+2v-1}$$

$$\Rightarrow x\frac{dv}{dx}=-\frac{(v^2+1)(v+1)}{v^2+2v-1}$$

$$\Rightarrow \frac{v^2-1+2v}{(v^2+1)(v+1)}dv=-\frac{dx}{x}$$

$$\Rightarrow \int\left(\frac{2v}{v^2+1}-\frac{1}{v+1}\right)dv+\int\frac{dx}{x}=0$$

$\Rightarrow \log(v^2+1)-\log(v+1)+\log x=\log c$

$$\Rightarrow \log x\frac{(v^2+1)}{v+1}=\log c$$

$$\Rightarrow \frac{y^2+x^2}{y+x}=c$$

$\Rightarrow x=-1, y=1 \quad \Rightarrow c=\infty$

So, $x+y=0$, which represents a straight line.

27. (d) $Ax^2+By^2=1$

$$\Rightarrow 2Ax+2By\frac{dy}{dx}=0$$

$$\Rightarrow \frac{y}{x}\frac{dy}{dx} = -\frac{A}{B}$$

$$\frac{1}{x^2}\left\{x\left[y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right] - y\frac{dy}{dx}\right\} = 0$$

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = y\frac{dy}{dx}$$

So, differential equation is of 2nd order and 2nd degree.

28. (a) $x^2\,dy - y(x + y)\,dx = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y(x+y)}{x^2} \qquad \text{...(i)}$$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Put in eq. (i),

So, $v + x\frac{dv}{dx} = v(1+v) = v + v^2$

$$\int \frac{dv}{v^2} = \int \frac{dx}{x}$$

$$\Rightarrow -\frac{1}{v} = \log x + \log c$$

$$\Rightarrow -\frac{x}{y} = \log x + \log c$$

or $-\frac{x}{y} = \log x\,c$ or $x = ke^{-x/y}$

29. (a) $\frac{dy}{dx} = \frac{y-1}{x^2+x}$

$$\Rightarrow \frac{dy}{y-1} = \frac{dx}{x^2+x} = \frac{dx}{x(x+1)}$$

$$\Rightarrow \int \frac{dy}{y-1} = \int\left(\frac{1}{x} - \frac{1}{x+1}\right)dx$$

$\Rightarrow \log(y - 1) = \log x - \log(x + 1) + \log c$

$(y - 1)(x + 1) = cx$

Passes through (1, 0)

$\Rightarrow c = -2$ or $(y - 1)(x + 1) + 2x = 0$

30. (b) $y\,dx + (x + x^2y)\,dy = 0$

$\Rightarrow y\,dx + x\,dy + x(xy)\,dy = 0$

$$\Rightarrow \frac{y\,dx + x\,dy}{x^2y^2} + \frac{dy}{y} = 0 \qquad \{\text{dividing by } x^2y^2\}$$

$$\Rightarrow -\frac{1}{xy} + \log y = c$$